

aicap

Associazione Italiana

Calcestruzzo Armato e Precompresso

Pisa

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Aula Magna dell'Università di Pisa
Facoltà di Ingegneria

GUIDA ALL'USO DELL'EUROCODICE 2 NELLA PROGETTAZIONE STRUTTURALE

STATI LIMITE DI ESERCIZIO (SEZ.7)

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STATI LIMITE DI ESERCIZIO

Il documento EC2 al punto 7.1 prende in considerazione i seguenti stati limite di esercizio

- Limitazione delle tensioni
- Controllo della fessurazione
- Controllo degli spostamenti

Lo stato limite di vibrazione, pur riconosciutane l'importanza per alcune particolari strutture, non è oggetto di trattazione nel documento

Lo stato limite di tensione in esercizio

- Le ragioni delle limitazioni delle tensioni
 - Calcestruzzo
 - Impedire fessure longitudinali negli elementi compressi in c.a.
 - Impedire microfessure e deformazioni viscosse non lineari
 - Garantire la durabilità strutturale
 - Acciaio
 - Impedire sforzi anelastici in esercizio
 - Impedire fessure troppo ampie
 - Garantire la durabilità strutturale

Lo stato limite di tensione in esercizio

■ Limitazione degli sforzi per il calcestruzzo

$$|\sigma_c| \leq k_1 f_{ck} \quad (\text{XD, XF, XS}) \quad |\sigma_c| \leq k_2 f_{ck} \quad \text{per impedire viscosità non lineare}$$
$$k_1 = 0.6 \quad k_2 = 0.45$$

■ Viscosità lineare

$$\Phi(t, t_0) = \Phi_0(t_0) \beta_c(t - t_0)$$

■ Viscosità non lineare

$$\Phi_{0,k}(t_0) = \Phi_0(t_0) \exp[\alpha_\sigma (k_\sigma - 0.4)] \quad 0.4 \leq k_\sigma < 0.6$$

$$\Phi_{0,k}(t_0) = \Phi_0(t_0) \quad k_\sigma \leq 0.4$$

$$k_\sigma = \frac{\sigma_c}{f_{cm}(t_0)}$$

$$\alpha_\sigma = 1.5$$

Lo stato limite di tensione in esercizio

■ Limitazioni tensionali per l'acciaio

$$\sigma_s \leq k_3 f_{yk} \quad k_3 = 0.8 \quad \text{Combinazione di carico caratteristica}$$

$$\sigma_s \leq k_4 f_{yk} \quad k_4 = 1 \quad \text{Deformazioni imposte}$$

$$\sigma_{sp} \leq k_5 f_{pk} \quad k_5 = 0.75 \quad \text{Acciaio da precompressione}$$

Lo stato limite di tensione in esercizio per sezioni in c.a. in assenza di fessurazione

$$\sigma_{ct} \leq f_{ct,eff}$$

$$f_{ct,eff} = \max \left[(1.6 - h \cdot 10^{-3}) \cdot f_{ctm} ; f_{ctm} \right]$$

$$f_{ctm} = 0.30 f_{ck}^{2/3} \quad (f_{ck} \leq 60 \text{ MPa})$$

$$f_{ctm} = 2.12 \ln \left[1 + \frac{(f_{ck} + 8)}{10} \right] \quad (f_{ck} > 60 \text{ MPa})$$

- Gli sforzi devono valutarsi nello stadio I non fessurato, assumendo come fattore di omogeneizzazione α_e l'espressione

$$\alpha_e = \frac{2 \cdot 10^5}{22 \cdot 10^3 \left[\frac{(f_{ck} + 8)}{10} \right]^{0.3}}$$

$$\alpha_e = 15$$

Per azioni istantanee

Per azioni permanenti

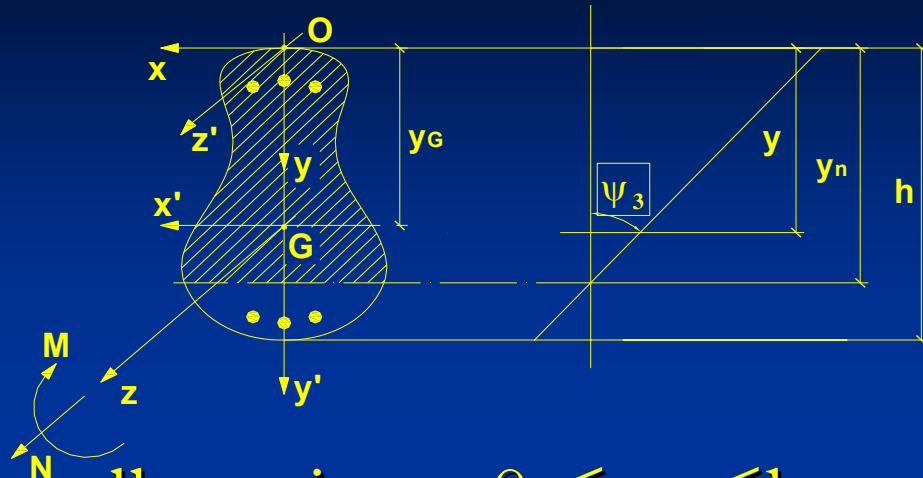
Equazioni generali per sezioni in c.a.

$$\sigma_c = E_c \varepsilon_c \quad (\varepsilon_c < 0)$$

$$\sigma_c = 0 \quad (\varepsilon_c > 0)$$

$$\sigma_s = E_s \varepsilon_s$$

$$\varepsilon_c = \varepsilon_s = \psi_3 (y - y_n)$$



■ **Asse neutro interno** alla sezione: $0 \leq y_n \leq h$

$$E_c \psi_3 \left[\int_0^{y_n} (y - y_n) b(y) dy + \alpha_e \sum_i (y_i - y_n) A_{si} \right] = N \quad E_c \psi_3 S_{yn}^* = N$$

$$E_c \psi_3 \left[\int_0^{y_n} (y - y_n)^2 b(y) dy + \alpha_e \sum_i (y_i - y_n)^2 A_{si} \right] = M + N (y_G - y_n) \quad E_c \psi_3 I_{yn}^* = M + N (y_G - y_n)$$

$$I_{yn}^* / S_{yn}^* = e + y_G - y_n \quad e = M / N$$

$$\psi_3 = [M + N (y_G - y_n)] / E_c I_{yn}^*$$

$$\sigma_c^{\min} = [M + N (y_G - y_n)] \times (-y_n) / I_{yn}^* \quad \sigma_s^{\max} = [\alpha_e (M + N (y_G - y_n))] \cdot (y_i^{\max} - y_n) / I_{yn}^*$$

Equazioni generali per sezioni in c.a.

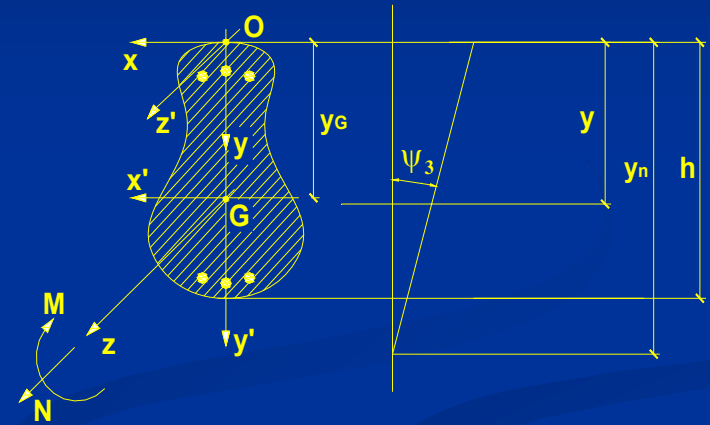
- Asse neutro esterno alla sezione: $y_n > h$

$$\varepsilon_c = \psi_3(y - y_n) \quad \bar{\sigma}_c = E_c \psi_3(y - y_n) \quad (0 \leq y \leq h) \quad \sigma_s = E_s \psi_3(y - y_n)$$

$$\bar{S}_{yn}^* = \left[\int_0^h (y - y_n) b(y) dy + \alpha_e \sum_i (y_i - y_n) A_{si} \right]$$

$$\bar{I}_{yn}^* = \left[\int_0^h (y - y_n)^2 b(y) dy + \alpha_e \sum_i (y_i - y_n)^2 A_{si} \right]$$

$$\bar{e} y_n = -r^2$$



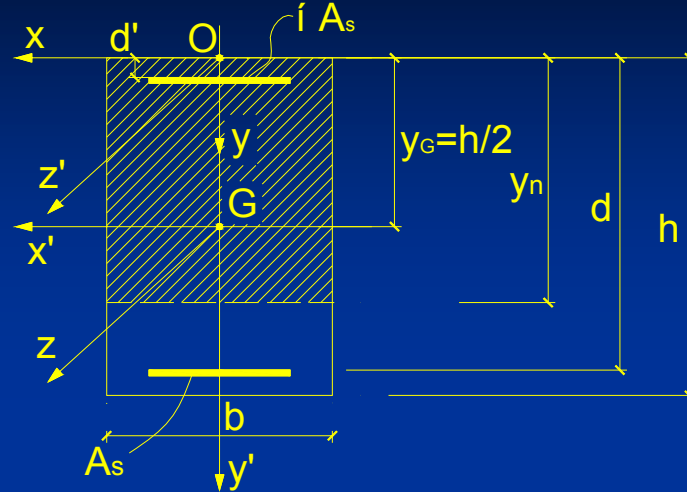
- Asse neutro esterno alla sezione: $y_n \leq 0$

$$\varepsilon_c > 0 \quad ; \quad \sigma_c = 0 \quad \sigma_s = E_s \psi_3(y - y_n) \quad (y_n \leq 0)$$

$$\bar{\bar{S}}_{yn} = \sum_i (y_i - y_n) A_{si} \quad \bar{\bar{I}}_{yn} = \sum_i (y_i - y_n)^2 A_{si}$$

$$\sigma_s^{\max} = \left[M + N(y_G - y_n) \right] \cdot (y_i^{\max} - y_n) / \bar{\bar{I}}_{yn} \quad \bar{\bar{e}} y_n = -r^2$$

Specializzazione per sezioni rettangolari



$$\frac{\bar{I}_{yn}^*}{\bar{S}_{yn}^*} = \frac{b y_n^3 / 3 + \alpha_e \cdot A_s \left[(d - y_n)^2 + \beta (d' - y_n)^2 \right]}{-b y_n^2 / 2 + \alpha_e \cdot A_s \left[(d - y_n) + \beta (d' - y_n) \right]} = e + \frac{h}{2} - y_n \quad 0 \leq y_n \leq h$$

$$\frac{\bar{I}_{yn}^*}{\bar{S}_{yn}^*} = \frac{b h^3 / 12 + b h (h/2 - y_n)^2 + \alpha_e \cdot A_s \left[(d - y_n)^2 + \beta (d' - y_n)^2 \right]}{b h (h/2 - y_n) + \alpha_e \cdot A_s \left[(d - y_n) + \beta (d' - y_n) \right]} = e + \frac{h}{2} - y_n \quad y_n \geq h$$

$$\frac{\bar{I}_{yn}^{\bar{\bar{}}}}{\bar{S}_{yn}^{\bar{\bar{}}}} = \frac{(d - y_n)^2 + \beta (d' - y_n)^2}{(d - y_n) + \beta (d' - y_n)} = e + \frac{h}{2} - y_n \quad y_n \leq 0$$

Stato limite di fessurazione

Prescrizioni per l'armatura minima

$$A_{s,min} \sigma_s = k_c k_f f_{ct,eff} A_{ct}$$

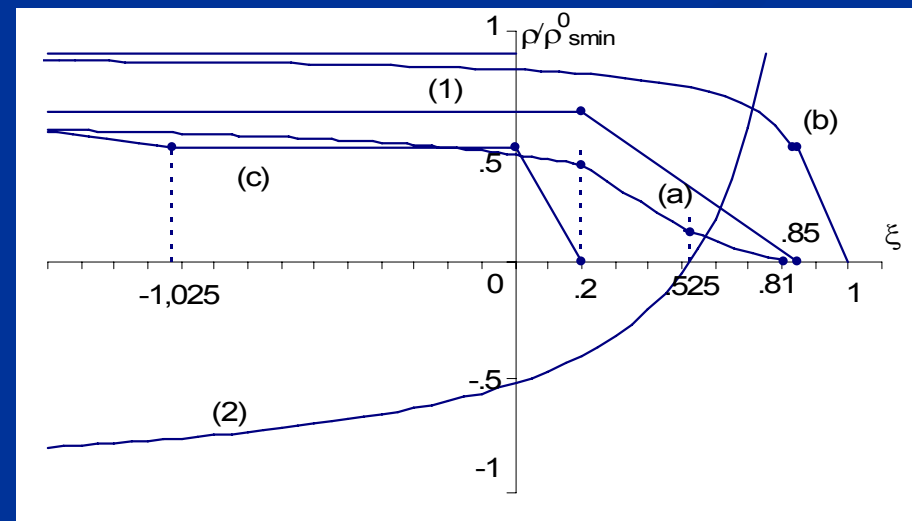
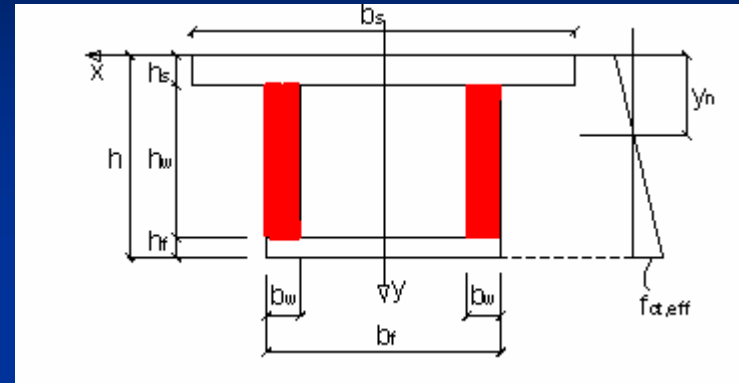
● $k_c = 0.4 \left[1 - \frac{\sigma_c}{k_1 \left(\frac{h}{h^*} \right) f_{ct,eff}} \right] \leq 1$

○ $k_c = 0.9 \frac{F_{cr}}{A_{ct} f_{ct,eff}} \geq 0.5$

$$\sigma_c = \frac{N_{E,d}}{bh}$$

$h^* = h$ per $h < 1.0$ m,
 $h^* = 1.0$ m per $h \geq 1.0$ m,

$k_1 = 1.5$ se $N_{E,d}$ è di compressione,
 $k_1 = 2h^*/3h$ se $N_{E,d}$ è di trazione,



$$\rho_{s,min}^0 = k_f f_{ct,eff} / \sigma_s \quad \alpha_s = 0.20; \alpha_f = 0.15; h/h^* = 1$$

Stato limite di fessurazione

$$\frac{\sigma_{s,cr}}{E_s} \leq \frac{\sigma_s}{E_s} \leq 2.5 \frac{\sigma_{s,cr}}{E_s}$$

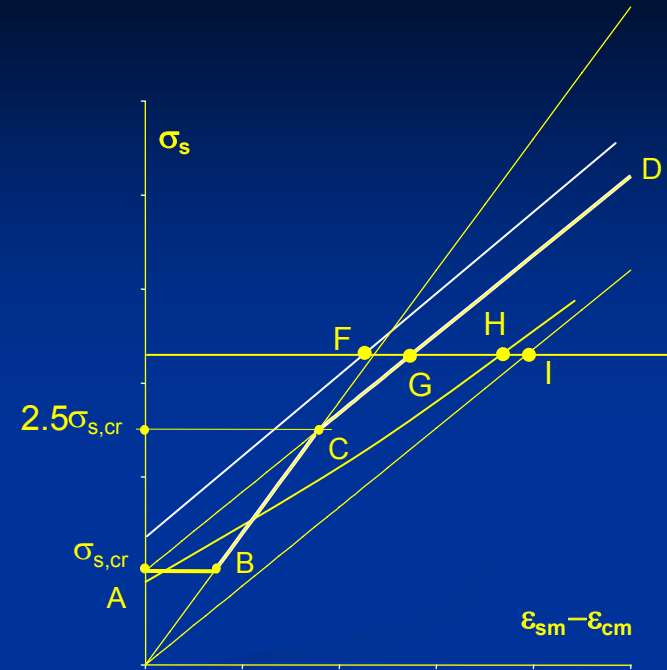
$$\Delta_\varepsilon = 0.6 \frac{\sigma_s}{E_s}$$

$$\sigma_s \geq 2.5 \sigma_{s,cr}$$

$$\Delta_\varepsilon = \frac{\sigma_s}{E_s} \left(1 - \frac{\sigma_{s,cr}}{\sigma_s} \right)$$

$$w_k = \bar{\beta} \frac{\sigma_s}{E_s} \left(1.5c + 0.04 \frac{\phi_s}{\rho_s} \right) \left(1 - \frac{\sigma_{scr}}{\sigma_s} \right) \quad (\text{Ferry Borges})$$

$$\bar{\beta} = 1.66 \quad \sigma_{scr} = \frac{0.75}{\rho_s}$$



Effetto di irrigidimento
del calcestruzzo

FI(F.B.)

GI(EC2 2004)

HI(EC2 ENV)

$$w_k = \bar{\beta} \frac{\sigma_s}{E_s} \left(50 + k_1 k_2 k_4 \frac{\lambda \phi_s}{\rho_s} \right) \left(1 - \beta \left(\frac{\sigma_{scr}}{\sigma_s} \right)^2 \right) \quad (\text{EC2, ENV1992})$$

$$\bar{\beta} = 1.7 \quad \beta \leq 1$$

$$w_k = \frac{\sigma_s}{E_s} \left(3.4c + k_1 k_2 k_4 \frac{\lambda \phi_s}{\rho_s} \right) \left(1 - \frac{\sigma_{scr}}{\sigma_s} \right) \quad (\text{EC2, EN1992 : 2004E})$$

$$\sigma_{scr} = 0.15 f_{ctm} / \rho_s + k_t \alpha_e f_{ctm} \quad k_t = 0.6 \quad k_4 = 0.425$$

Formule di progetto

■ Procedimento generale

$$-\frac{1}{2}\xi^2 - \alpha_e \cdot \rho_s (1+\beta)\xi + \alpha_e \cdot \rho_s (\delta + \beta \cdot \delta') = 0 \quad v = \frac{M}{M_{cr}} = \frac{M}{k_t f_{ctm} b \cdot h^2 / 6} \quad \sigma_s = \frac{\alpha_e v (\delta - \xi) f_{ctm} k_t}{2 \left[3\alpha_e \cdot \rho_s \left[(\delta - \xi)^2 + \beta (\delta' - \xi)^2 \right] + \xi^3 \right]}$$

$$\rho_s = \frac{\xi^2}{2\alpha_e \left[-(1+\beta)\xi + \delta + \beta\delta' \right]} \quad p = \sigma_s / k_t f_{ctm} \quad \frac{\alpha_e v (\delta - \xi) - 2\xi^3 p}{(\delta - \xi)^2 + \beta (\delta' - \xi)^2} = \frac{3\xi^2 p}{\delta + \beta\delta' - (1+\beta)\xi}$$

$$p = \frac{\bar{w}_k^0}{3.4 \cdot c + 0.17 \varphi \cdot \lambda / \rho_s} + \frac{\lambda}{\rho_s} + \alpha_e \quad \bar{w}_k^0 = E_s \bar{w}_k / k_t f_{ctm}$$

$$\frac{\alpha_e v (\delta - \xi)}{\left[(\delta - \xi)^2 + \beta (\delta' - \xi)^2 \right]} = \left[\frac{\bar{w}_k^0 \cdot \xi^2}{3.4 \cdot c \cdot \xi^2 + 0.34 \alpha_e \cdot \varphi \cdot \lambda \left[\delta + \beta \delta' - (1 + \beta) \xi \right]} + \frac{2\alpha_e \times \lambda}{\xi^2} \left[\delta + \beta \delta' - (1 + \beta) \xi \right] + \alpha_e \right] \cdot \left[\frac{3\xi^2}{\delta + \beta \delta' - (1 + \beta) \xi} + \frac{2\xi^3}{(\delta - \xi)^2 + \beta (\delta' - \xi)^2} \right]$$

Formule di progetto

■ Procedimento approssimato

$$\bar{W}_k = \frac{\sigma_s}{E_s} \left(3.4c + 0.17 \frac{\phi \cdot \lambda}{\rho_s} \right) \left[1 - \frac{\lambda}{\rho_s \cdot p} \left(1 + \frac{\alpha_e \cdot \rho_s}{\lambda} \right) \right]$$

$$h_0 = 0.9d; \quad \xi = 0.3\delta. \quad \lambda = \frac{(1-0.3\delta)}{3} \quad \sigma_s A_s \cdot 0.9d = M \quad \rho_s = \frac{0.185 v}{p \cdot \delta}$$

$$v^* = \frac{v}{1 - \frac{\delta \cdot \lambda}{0.185v}} = \frac{v}{1 - \frac{1.18}{v}} \quad u_1 = \frac{c}{\phi}$$

$$u_2 = \frac{\bar{W}_{0k}}{\phi}$$

$$p^2 + 5 \cdot v^* \left[3.4u_1 \frac{v}{v^*} - 0.20 \frac{\alpha_e}{v} \right] p - v^* [17\alpha_e \cdot u_1 + 5u_2] = 0$$

noti v, \bar{W}_k, ϕ \rightarrow σ_s, ρ_s

$$\phi = \frac{v \left[\frac{v^*}{v} \cdot (17c_e + 5\bar{W}_{0k}) - 17cp \right]}{p^2 - p\alpha_e \frac{v^*}{v}}$$

noti v, \bar{W}_k, p

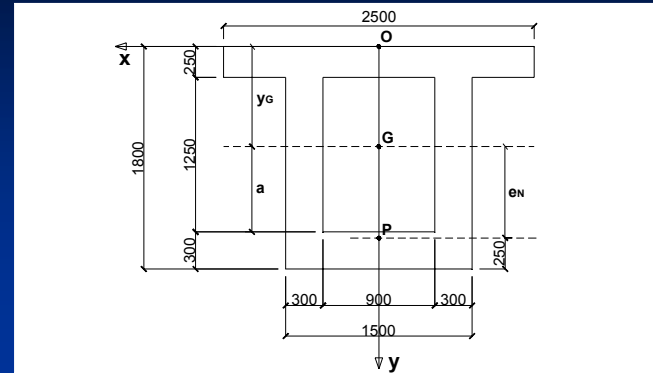
ϕ, ρ_s

$$\rho_s p = 0.20v$$

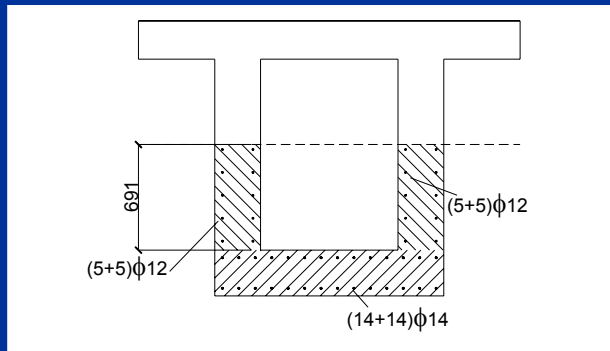
Esempio 1

Calcolo dell'armatura minima

- 1) $M = M_{cr}$; $N = 0$
- 2) $N = -6000$ kN ; $e_N = 741$ mm

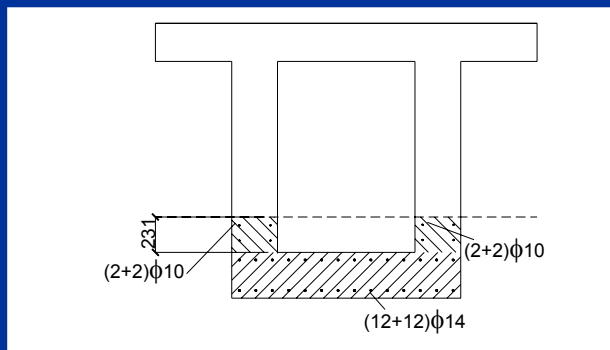


$$f_{ck} = 45 \text{ MPa}; \quad f_{ct,eff} = 3.8 \text{ MPa}; \quad \sigma_s = 200 \text{ MPa}; \quad k = 0.65 \quad (h_w > 1\text{m})$$



1)

$$a = 691 \text{ mm}$$



2)

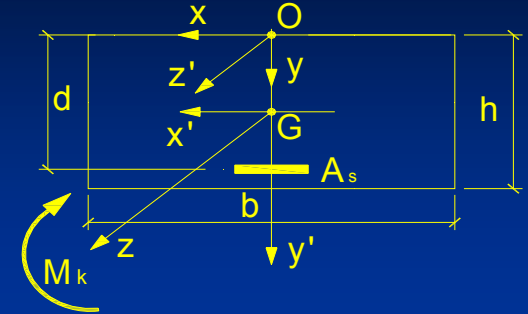
$$M_{cr} = \left[-\frac{N}{A} \left(1 + e_N \frac{A}{W_i} \right) + f_{ct,eff} \right] W_i$$

$$y_n = y_G - \frac{r^2}{e} \quad e = \frac{M_{cr} + N \cdot e_N}{N}$$

$$y_n = 1269 \text{ mm} \Rightarrow a = 231 \text{ mm}$$

Esempio 2

- Controllo della fessurazione,
 - Progetto con noti Φ , ν \bar{w}_k , applicazione del procedimento approssimato



$$\bar{w}_k = 0.1 \text{ mm} \quad ; \quad \bar{w}_k = 0.2 \text{ mm} \quad ; \quad \bar{w}_k = 0.3 \text{ mm}$$

$$b = 100 \text{ cm} \quad ; \quad h = 50 \text{ cm} \quad ; \quad c = 5 \text{ cm} \quad f = 26 \text{ mm} \quad ;$$

$$f_{ck} = 33 \text{ MPa} \quad ; \quad k_t = 0.6 \quad \alpha_e = 15 \quad ; \quad M_k = 600 \text{ kNm}$$

$$f_{ctm} = 0.3 \cdot 33^{2/3} = 3.086 \text{ MPa} \quad ; \quad \delta = (50 - 6.3) / 50 = 0.874$$

$$M_{cr} = 0.6 \cdot 3.086 \cdot (100 \cdot 50^2 / 6) \cdot 10^3 \cdot 10^{-6} = 77.15 \text{ kNm}$$

$$\nu = 600 / 77.15 = 7.77 \quad ; \quad \nu^* = 7.77 / (1 - 1.18 / 7.77) = 9.16 \quad ; \quad u_1 = 50 / 26 = 1.92$$

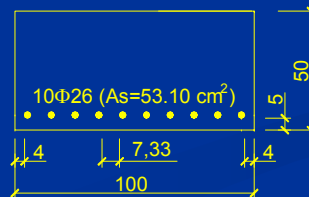
$$\bar{w}_{k_i} = \bar{w}_k^{\max} \cdot k_{w_i} \quad k_{w_i} = i/3 \quad (i=1, 2, 3) \quad \bar{w}_{0k_i} = \frac{2 \cdot 10^5 \cdot 0.3}{0.6 \cdot 3.086} \cdot k_{w_i} = 32404 \cdot k_{w_i} \quad u_2 = \frac{32404}{26} \cdot k_{w_i} = 1246 \cdot k_{w_i}$$

$$p^2 + 5 \cdot \nu^* \left[3.4 u_1 \frac{\nu}{\nu^*} - 0.20 \frac{\alpha_e}{\nu} \right] p - \nu^* [17 \alpha_e \cdot u_1 + 5 u_2] = 0$$

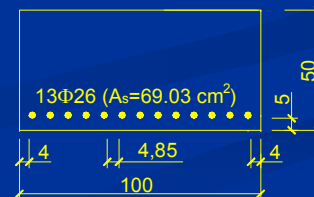
$$\rho_s p = 0.20 \nu$$

$$p = \sigma_s / k_t f_{ctm}$$

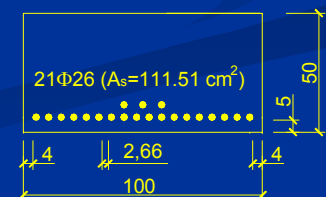
$$p^2 + 235.93 p - 4485 - 57067 k_w = 0$$



C



B



A

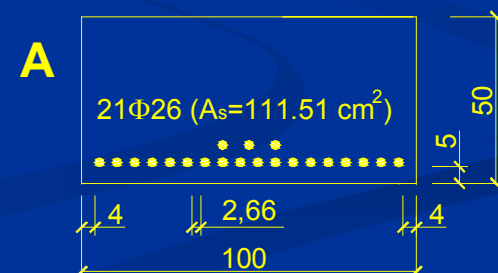
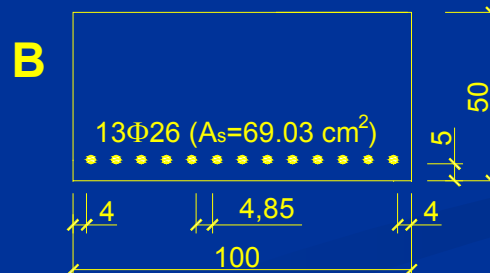
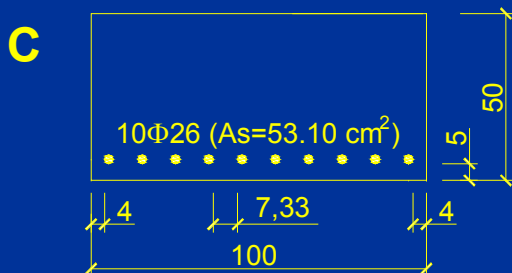
Esempio 2

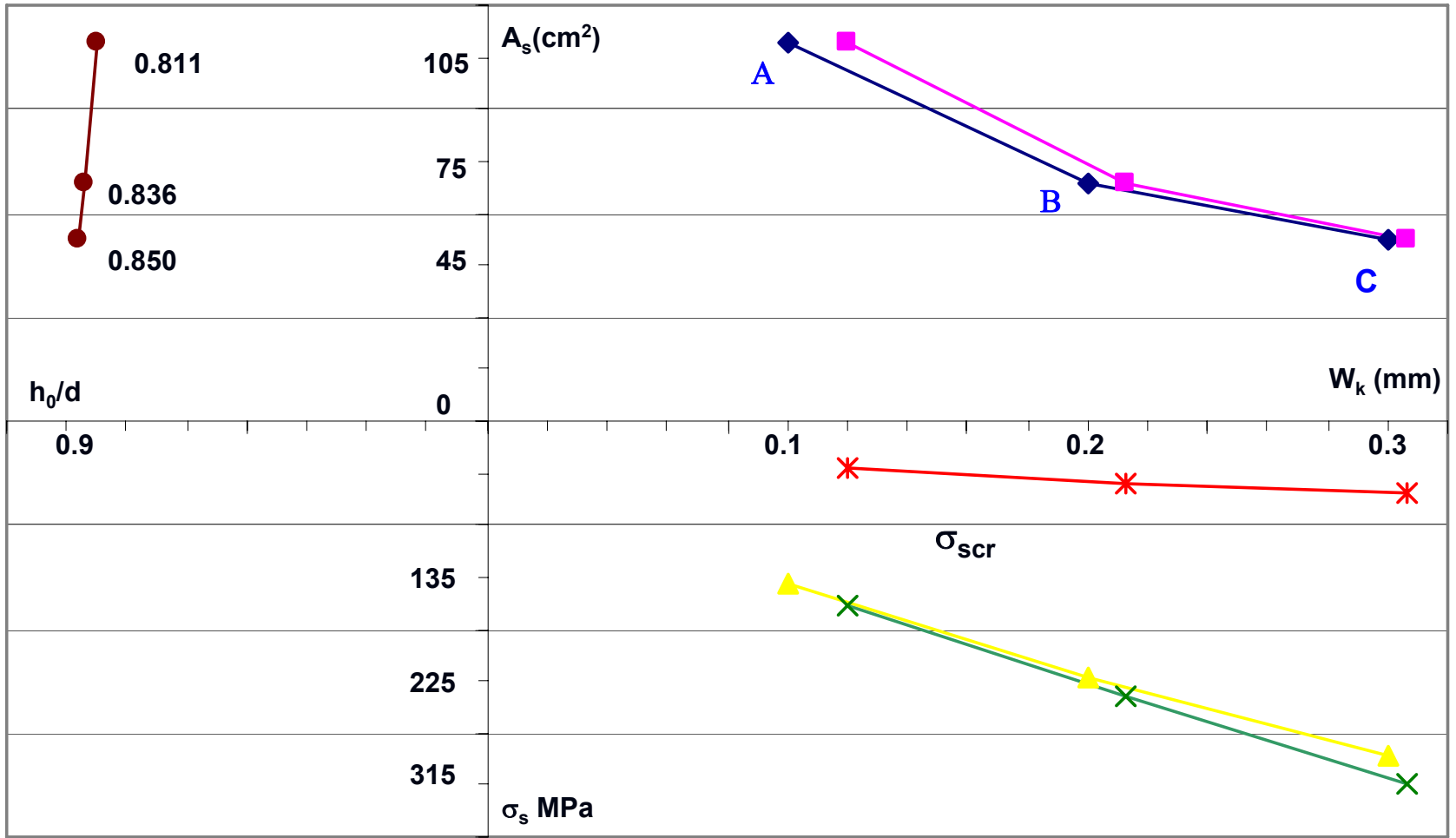
- Stato limite di fessurazione
 - Verifiche secondo la formulazione generale

$$-\frac{1}{2}\xi^2 - \alpha_e \cdot \rho_s (1+\beta)\xi + \alpha_e \cdot \rho_s (\delta + \beta \cdot \delta') = 0$$

$$\sigma_s = \frac{\alpha_e v (\delta - \xi) f_{ctm} k_t}{2 \left[3\alpha_e \cdot \rho_s \left[(\delta - \xi)^2 + \beta (\delta' - \xi)^2 \right] + \xi^3 \right]}$$

$$p = \sigma_s / k_t f_{ctm} \quad \bar{w}_{0k} = (p - \lambda / \rho_s - \alpha_e) (3.4c + 0.17\phi\lambda / \rho_s) \quad \bar{w}_k^0 = E_s \bar{w}_k / k_t f_{ctm}$$





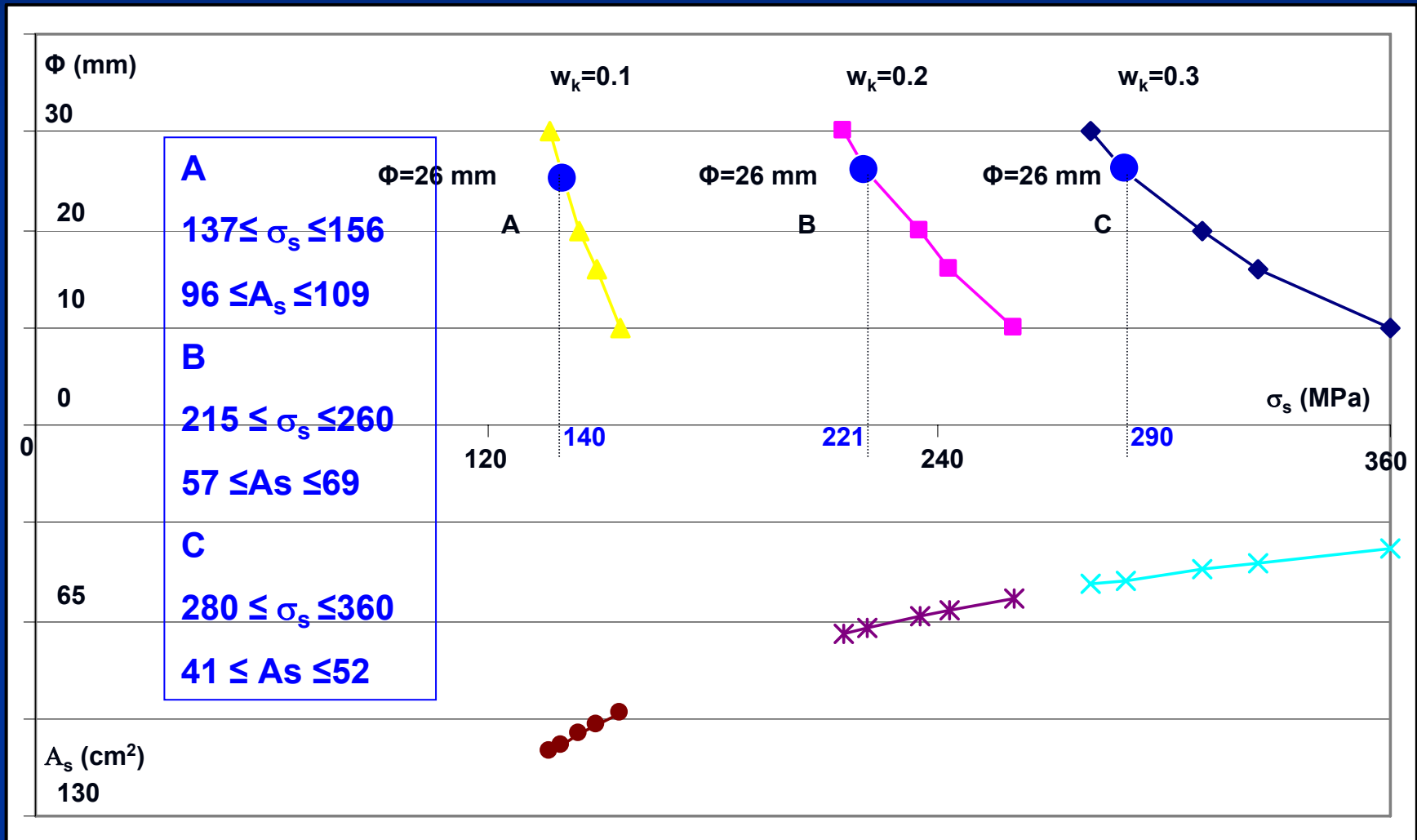
w_k (mm)	A_s (mm ²)	S_s (MPa)	h_0/d
0.1	11151	140	0.9
0.2	6903	221	0.9
0.3	5310	190	0.9

w_k (mm)	A_s (mm ²)	S_s (MPa)	h_0/d
0.120	11151	160	0.811
0.213	6903	238	0.836
0.306	5310	304	0.850

Esempio 2

Determinazione del massimo diametro

$$\phi_{\max} = \frac{[17c(v_p - \alpha_e v^*) - 5v^* \bar{w}_{ok}]}{\alpha_e v^* / v_p - p^2}$$

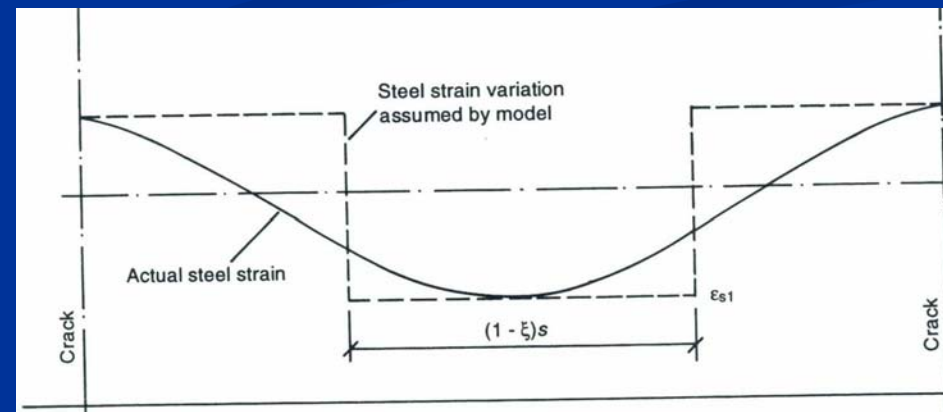
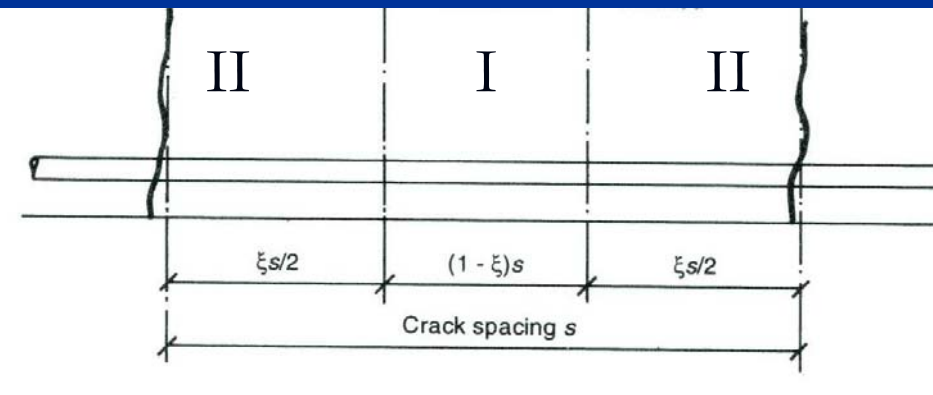


Stato limite di deformazione

■ Relazioni generali

$$\chi = \chi_1 (1 - \zeta) + \chi_{II} \zeta$$

$$\zeta = 1 - \beta \left(\sigma_{s,r} / \sigma_s \right)^2$$



Stato limite di deformazione

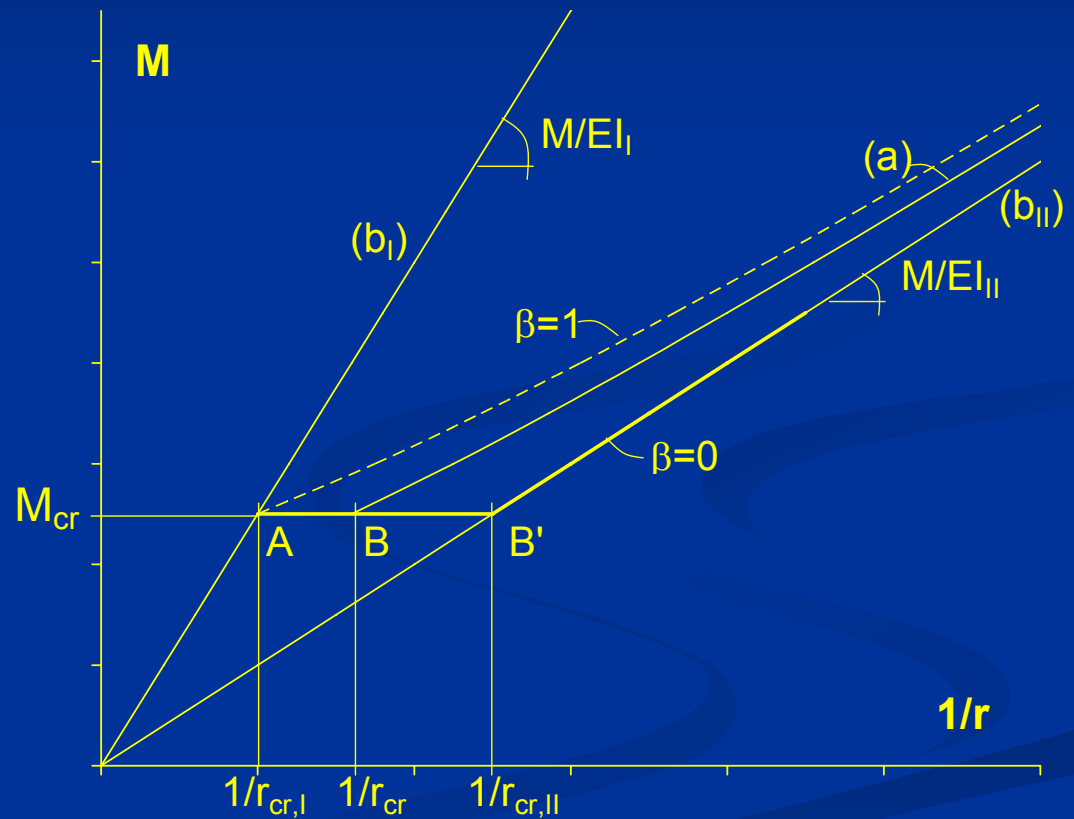
■ Diagramma Momenti-Curvatura

$$\frac{1}{r} = \frac{1}{r_I}(1-\zeta) + \frac{1}{r_{II}}\zeta$$

$$\zeta = 1 - \beta(\sigma_{s,r}/\sigma_s)^2 \quad \zeta = 1 - \beta(M_{cr}/M)^2$$

$$\frac{1}{r_I} = \frac{M}{E_c I_I} \quad \frac{1}{r_{II}} = \frac{M}{E_c I_{II}}$$

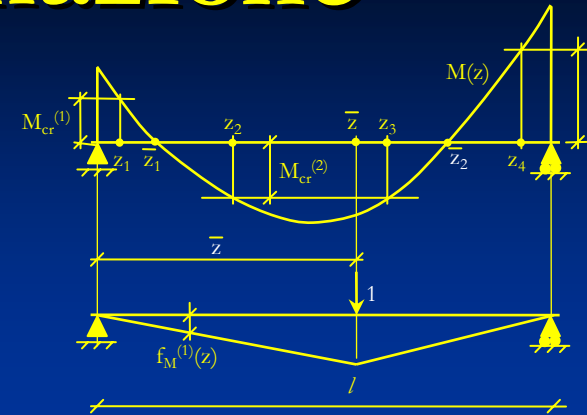
$$\frac{1}{r} = (M/EI_I)[1 + (c-1)\zeta] \quad c = \frac{I_I}{I_{II}}$$



Stato limite di deformazione

■ Calcolo degli abbassamenti

$$\frac{1}{r} = \frac{M}{EI_i} [1 + (c-1)\zeta]$$



$$\frac{1}{r_1} = \frac{M}{EI_{i,1}} [1 + (c_1 - 1)\zeta_1]$$

$$0 \leq z \leq z_1$$

$$\frac{1}{r_5} = \frac{M}{EI_{i,5}}$$

$$z_3 \leq z \leq \bar{z}_2$$

$$\frac{1}{r_2} = \frac{M}{EI_{i,2}}$$

$$z_1 \leq z \leq \bar{z}_1$$

$$\frac{1}{r_6} = \frac{M}{EI_{i,6}}$$

$$\bar{z}_2 \leq z \leq z_4$$

$$\frac{1}{r_3} = \frac{M}{EI_{i,3}}$$

$$\bar{z}_1 \leq z \leq z_2$$

$$\frac{1}{r_7} = \frac{M}{EI_{i,7}} [1 + (c_7 - 1)\zeta_7]$$

$$z_4 \leq z \leq l$$

$$\frac{1}{r_4} = \frac{M}{EI_{i,4}} [1 + (c_4 - 1)\zeta_4]$$

$$z_2 \leq z \leq z_3$$

$$\text{with } I_{i,1} = I_{i,2} \quad ; \quad I_{i,3} = I_{i,4} = I_{i,5} \quad ; \quad I_{i,6} = I_{i,7}$$

$$v(\bar{z}) = \sum_{i=1}^7 \int_{\ell_i} f_M^{(1)}(z, \bar{z}) \frac{1}{r_i(z)} dz$$

$$v(\bar{z}) = \sum_{i=1}^7 \int_{\ell_i} f_M^{(1)}(z, \bar{z}) \frac{M(z)}{EI_{i,j}} dz + \sum_{j=1,4,7} \int_{\ell_j} f_M^{(1)}(z, \bar{z}) \frac{M(z)}{EI_{i,j}} (c_j - 1) \left(1 - \beta \frac{M_{cr,j}^2}{M^2} \right) dz \quad v(\bar{z}) = v_1(\bar{z}) + \Delta v(\bar{z})$$

$$v_1(\bar{z}) = \sum_{i=1}^7 \int_{\ell_i} f_M^{(1)}(z, \bar{z}) \frac{M(z)}{EI_{i,j}} dz$$

$$\Delta v(\bar{z}) = \sum_{j=1,4,7} \int_{\ell_j} f_M^{(1)}(z, \bar{z}) \frac{M(z)}{EI_{i,j}} (c_j - 1) \left(1 - \beta \frac{M_{cr,j}^2}{M^2} \right) dz$$

Esempio 3

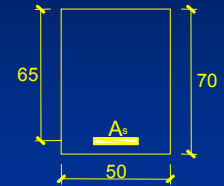
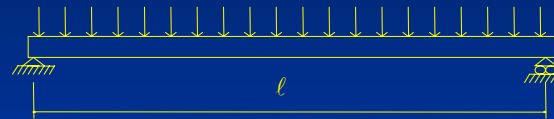
$$f_{ck}=30 \text{ MPa} ; q=4 \text{ kN/m} ; l=10 \text{ m} ; A_s=31.64 \text{ cm}^2 (7\phi 24) ; \alpha_e=15$$

$$A^* = 70 \cdot 50 + 15 \cdot 31.64 = 3974.6 \text{ cm}^2$$

$$y_G^* = \frac{70 \cdot 50 \cdot 35 + 15 \cdot 31.64 \cdot 65}{3974.6} = 38.58 \text{ cm}$$

$$I^* = \frac{50 \cdot 70^3}{12} + 50 \cdot 70 \cdot 3.58^2 + 15 \cdot 31.64 \cdot (65 - 35.58)^2 = 1805303 \text{ cm}^4$$

$$W_i^* = \frac{1805303}{70 - 38.58} = 57457 \text{ cm}^3$$



$$f_{ctm} = 0.30 \cdot 30^{2/3} = 2.9 \text{ MPa}$$

$$M_{cr} = f_{ctm} W_i^* = 2.9 \cdot 57457 \cdot 10^{-6} = 166.6 \text{ kNm}$$

$$M_{max} = 40 \cdot 10^2 / 8 = 500 \text{ kNm}$$

$$-50 \cdot \frac{y_n^2}{2} + 15 \cdot 31 \cdot (65 - y_n) = 0 \rightarrow y_n = -9.492 + \sqrt{9.492^2 + 1234} = 26.90 \text{ cm}$$

$$I_{II}^* = 50 \cdot \frac{26.90^3}{3} + 15 \cdot 31.64 \cdot (65 - 26.5)^2 = 1013352 \text{ cm}^4$$

$$c = I_1 / I_{II} = 1.78$$

$$\frac{1}{r} = \frac{M}{E_c I_1} , M \leq M_{cr} \quad \frac{1}{r} = \frac{M}{E_c I_1} \left[1 + \beta \left(\frac{M_{cr}}{M} \right)^2 (c-1) \right] , M \geq M_{cr} \quad v\left(\frac{l}{2}\right) = v_1\left(\frac{l}{2}\right) \cdot \left(1 + \frac{\Delta v(l/2)}{v_1(l/2)} \right)$$

$$\Delta v\left(\frac{l}{2}\right) = 2(c-1) \frac{M_{max} l^2}{E_c I_1^*} \left[\int_{\xi_1}^{1/2} f_M^{(1)}\left(\xi, \frac{l}{2}\right) g(\xi) d\xi - \beta \frac{M_{cr}^2}{M_{max}^2} \cdot \int_{\xi_1}^{1/2} f_M^{(1)}\left(\xi, \frac{l}{2}\right) (g(\xi))^{-1} d\xi \right]$$

Esempio 3

■ Stato limite di deformazione

■ Carico uniformemente distribuito

$$g(\xi) = 4(\xi - \xi^2); \quad f_M^{(1)}(\xi, 1/2) = \xi/2$$

$$\Delta v\left(\frac{l}{2}\right) = (c-1) \frac{M_{\max} l^2}{E_c I_1^*} \left[\int_{\xi_1}^{\frac{1}{2}} 4(\xi^2 - \xi^3) d\xi - \frac{\beta}{4\mu^2} \int_{\xi_1}^{\frac{1}{2}} \frac{d\xi}{1-\xi} \right]$$

$$\Delta v\left(\frac{l}{2}\right) = (c-1) \frac{M_{\max} l^2}{E_c I_1^*} \left[\frac{5}{48} + \xi_1^4 - \frac{4}{3} \xi_1^3 - \frac{\beta}{4\mu^2} \ln[2(1-\xi_1)] \right]$$

$$4(\xi_1 - \xi_1^2) = M_{cr}/M_{\max} = 1/\mu \quad \xi_1 = \frac{1}{2} \left[1 - \sqrt{\frac{\mu-1}{\mu}} \right] \quad v_1 = \frac{5}{48} \frac{M_{\max} l^2}{E_c I_1^*}$$

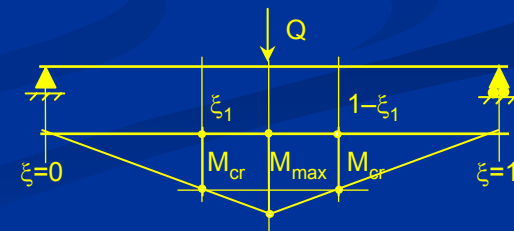
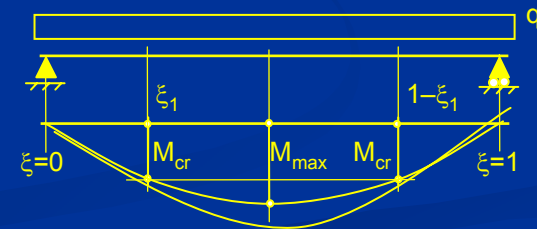
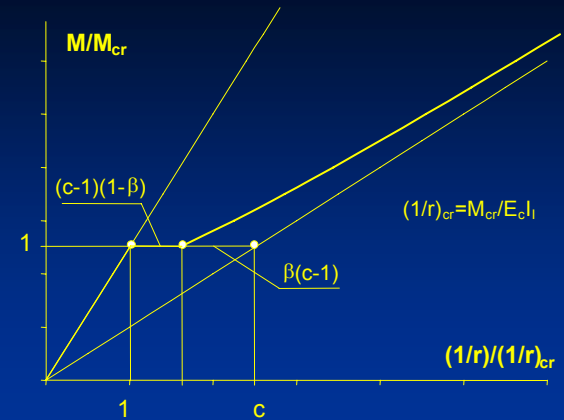
■ Carico concentrato

$$g(\xi) = \xi/2; \quad f_M^{(1)}(\xi, 1/2) = \xi/2$$

$$v\left(\frac{l}{2}\right) = \frac{5}{48} \frac{M_{\max} l^2}{E_c I_1^*} \left\{ 1 + (c-1) \left[1 + \frac{48}{5} \left(\xi_1^4 - \frac{4}{3} \xi_1^3 \right) - \frac{12}{5} \frac{\beta}{\mu^2} \ln[2(1-\xi_1)] \right] \right\}$$

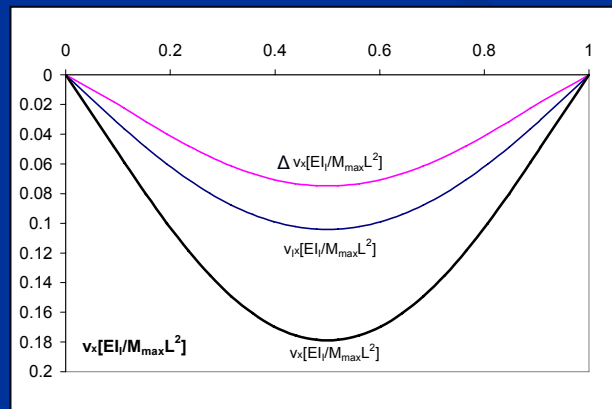
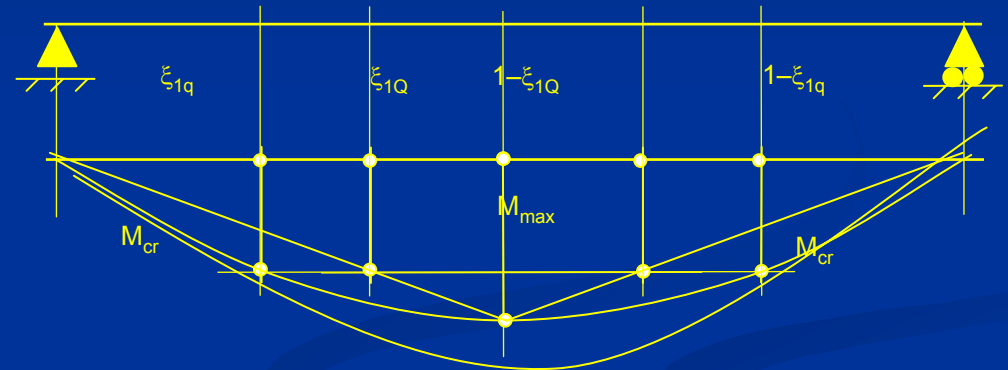
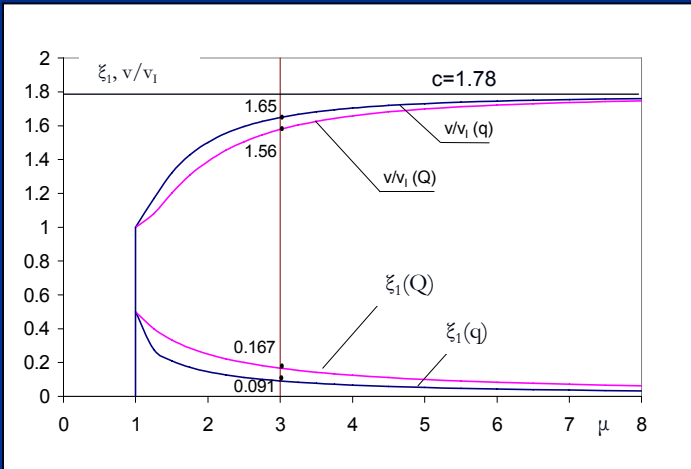
$$v\left(\frac{l}{2}\right) = \frac{M_{\max} l^2}{12 E_c I_1^*} \left[1 + (c-1) \left[1 - 8\xi_1^3 - \frac{3\beta}{\mu^2} (1-2\xi_1) \right] \right]$$

$$\xi_1 = \frac{1}{2\mu}$$



Esempio 3

■ Stato limite di deformazione



Conclusioni

- I modelli proposti da EC2 ENV 1992-1-1 E2004 per le analisi allo stato limite di esercizio introducono i seguenti concetti
 - La formulazione di s_{rm} che fornisce direttamente il valore caratteristico di apertura della fessura
 - L'adozione del contributo irrigidente del calcestruzzo indipendente dalla tensione dell'acciaio
- Le formule proposte da EC2 sono di semplice utilizzo per la valutazione dell'apertura delle fessure. Non sono altrettanto idonee per il progetto sotto prescritta ampiezza fessurativa
- Possono tuttavia derivarsi formule approssimate, basate su una prefissata posizione dell'asse neutro, che permettono una progettazione agile e sufficientemente approssimata delle sezioni in c.a. allo stato limite di fessurazione.